The *G* Proof™

Rigorous Mathematical Proof That God Exists

Using First-Order Predicate Logic
And Axioms That No Scientist Can Reasonably Deny

G Theory Version 2

Theory of Absolute Phenomena

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“The fear of infinity is a form of myopia that destroys the possibility of seeing the actual infinite, even though it in its highest form has created and sustains us, and in its secondary transfinite forms occurs all around us and even inhabits our minds.”

— Georg Cantor, 1845-1918 [1 at p. 43 of Rucker]

Contents

Abstract

Version 1 of *G* Theory is presented in Nine Videos at the website, TheGProof.org. Using five axioms, A1 through A5, and First Order Predicate Logic, the existence of a unique, self-causing, omnipotent phenomenon is proven. However, skeptics may question whether some of axioms A1-A5 accurately depict reality. In the spirit of skepticism, Version 2 of *G* Theory is presented here, entitled the Theory of Absolute Phenomena (TAP). TAP excludes A1, includes A2 and A3, modifies A4 and A5, and justifies 5 additional axioms, A6 through A10. This new theory features a second *G* proof and a modified definition for God, giving a slightly different result or "flavor" of God. A variation of TAP, which revises A1 and excludes some of the other axioms, also proves the same result.

TAP is consistent with the theory presented in Version 1, *i.e.* they may both be true; and if you happen to accept all of the axioms of Version 1, then you will probably also accept the axioms of TAP. On the other hand, some may believe that there exist Absolute (either self-causing, or entirely causeless) phenomena other than God; while Version 1 prohibits this, TAP specifically addresses this possibility.

The Causal Equivalence Principle demonstrates that in TAP, the question of whether Absolute phenomena should be self-causing or causeless is irrelevant because it bears no substantive consequences either way.

In order to maintain the philosophical position of atheism, rejections of Version 1, TAP, and its variation are all required. The logical consequences of these rejections are categorized such that any atheist must believe in something bizarre or impossible from one of a small number of categories.

Rules of Logic

G Theory and the proof that God exists rest on First Order Predicate Logic, where the notation (except for some symbols) and method of proof are taken from Kalish and Montague [6], with some modifications to improve presentation and clarity.

Lower case letters are variables. Capital letters, with certain exceptions such as G and H, are predicates. Also, certain symbols, including the equals sign, are predicates. As explained in Videos 3, 4 and 5 [2], the following symbols are used, and with the following meanings:

$\neg$ means NOT

$\wedge$ means AND

$\vee$ means OR

$\Rightarrow$ means IF...THEN (IMPLIES)

$\Leftrightarrow$ means IF AND ONLY IF

$\forall$ means FOR EVERY

$\exists$ means THERE EXISTS

$\exists!$ means THERE EXISTS EXACTLY ONE

ι means THE

The rules of logic used in the proofs are presented and explained in Videos 3, 4 and 5 [2], and are listed without explanation below. Additional rules of logic (omitted from the Videos) are included here with a brief description of their meaning.

Inference Rules

R: Repitition (Video 3)

MP: Modus Ponens (Video 3)

Simp: Simplification (Video 3)

Adj: Adjunction (Video 3)

BC: Biconditional to Conditional (Video 3)

CB: Conditional to Biconditional (Video 3)

UI: Universal Instantiation (Video 4)

EG: Existential Generalization (Video 4)

EI: Existential Instantiation (Video 4)

SE: Substitution of Equals (Video 4)

RE: Reflexivity of Equals (Video 5)

MUFS: Medium Uniqueness Formula to Short (Video 5)

SUFM : Short Uniqueness Formula to Medium (Video 5)

DN: Double Negation. This is the simplest rule of logic:

> From $\neg\neg P$,
> Infer P.

And vice-versa:

> From P,
> Infer $\neg\neg P$.

MT: Modus Tollens. This is the negated reverse of MP.

> From $P \Rightarrow Q$ and $\neg Q$,
> Infer $\neg P$.

MT is easily verified using IP (see below): to prove $\neg P$, assume P (by AIP, see below). Then Q follows by MP, which contradicts $\neg Q$.

LEM: Law of Excluded Middle. Any symbolic sentence is either true or false:

$$P \vee \neg P$$

This is a theorem of logic, *i.e.* a tautology.

SC: Separation of Cases. This rule requires a disjunction of 2 (or potentially more) possibilities, where each disjunct on its own (each of the "cases") implies the same conclusion:

> From $P \vee Q$, $P \Rightarrow R$, and $Q \Rightarrow R$,
> Infer R.

SC is especially useful in conjunction with LEM, where $\neg P$ is used for Q: if P is true, then R follows; and if P is false, then R still follows. Either way, R is true.

LL: Leibniz's Law. This may refer to various laws of equality, including Substitution of Equals. Since SE is already established in Video 4, LL is used for the opposite: if two things are known to differ with respect to some property, then they are unequal:

From Fx and ¬Fy,
Infer $x \neq y$.

LL is easily verified using IP (see below): to prove $x \neq y$, assume $x = y$ (by AIP, see below). Then Fy follows by SE, which contradicts ¬Fy.

Proof Starting and Ending Rules

Claim (Video 3)

DP: Direct Proof (Video 3)

(A)CP: (Assumption for) Conditional Proof (Video 3)

UP: Universal Proof (Video 4)

(A)IP: (Assumption for) Indirect Proof. This is known commonly as *reductio ad absurdum*. On the line following a Claim, the opposite is assumed. If a contradiction can then be proven, it means the assumption was false, and hence the Claim true. When this is accomplished, the done line cites the contradictory line numbers, the box is drawn, and "Prove" is canceled.

Ln	Statement	From	By
1	Prove P	—	Claim
2	¬P	—	AIP
...	...		
m	Q		
...	...		
n	¬Q		
n+1	done	m, n	IP

When a negation ¬P is the claim, its opposite P may be used as the assumption rather than ¬¬P.

Terminology

Three Categories of Phenomena

Consider phenomenon x. When we ask what causes x, disregarding any axioms, there are 3 logical possibilities:

1. x causes itself: x is *self-causing;*

2. another phenomenon causes x: x is *relative* (*i.e.* the existence of x is relative to the existence of something *else*); or

3. nothing causes x: x is *causeless*.

Formally, the following 3 definitions express these possibilities:

Def. Self-Causing

$$Sx \Leftrightarrow x \blacktriangleright x.$$

Def. Relative

$$Rx \Leftrightarrow \exists y\, (y \blacktriangleright x \wedge x \neq y).$$

Def. Causeless

$$Lx \Leftrightarrow \neg\exists y (y \blacktriangleright x).$$

In Version 1, Axiom 1 precisely excludes the possibility of Lx, forcing every phenomenon to be either self-causing or relative. Incidentally, it turns out that A1 is not required to prove the existence of God. TAP excludes A1 and allows causeless phenomena.

Consider whether it is realistic for a phenomenon to be both relative and self-causing. What do you think? If you think not, then you accept the Self-Causation Exclusion Principle:

Self-Causation Exclusion Principle (SCEP)

SCEP. $\neg\exists x\, (Sx \wedge Rx)$

No phenomenon is both self-causing and relative.

SCEP could reasonably be taken as an axiom, but it is not needed to prove any of the theorems presented herein. It is still a useful principle to consider when interpreting and justifying the axioms.

Absolute vs. Relative Phenomena

The changing world consists of phenomena which are created and destroyed. They exist relative to dimensions such as time and space. In *G* Theory, these are the *relative* phenomena, as defined above.

The unchanging world consists of Absolute Truth, or *Absolute* phenomena. This is the exact opposite of relative:

<u>Def. Absolute</u>

$\forall x(Ax \Leftrightarrow \neg\exists y(y \blacktriangleright x \wedge x \neq y))$. Or simply: $\neg Rx$.

As illustrated above, there are only 3 possibilities, so if x is Absolute rather than relative, then the remaining possibilities are either Sx or Lx, *i.e.* self-causing or causeless.

In Version 1, since Lx is prohibited, Absolute phenomena are identified as *self-causing.* However, for the sake of those who think it is possible that no phenomenon causes itself, Absolute phenomena are allowed to be causeless rather than self-causing. As it turns out, it is unnecessary to choose between Sx and Lx for any Absolute x, because:

1. TAP is completely neutral with respect to this choice, as will be demonstrated with the Causal Equivalence Principle in a later section, and

2. it may be impossible to verify empirically whether a phenomenon causes itself.

In order to ensure this neutrality, Cx is now redefined to mean that x causes something *other than itself*. In Version 1, since every Absolute phenomenon is self-causing, it follows from the original definition of C that every Absolute phenomenon causes something – namely itself. But since Absolute phenomena are now allowed to be causeless, it is conceivable that an Absolute phenomenon could cause nothing at all.

<u>Def. Causes Something Else</u>

$\forall x\ (Cx \Leftrightarrow \exists y\ (x \blacktriangleright y \wedge y \neq x))$

This new definition is the dual of relative: whenever $a \blacktriangleright b$ and $a \neq b$, it follows that Ca and Rb. If some phenomenon is known to have either property, then there must exist a different phenomenon which has the dual property.

Omnipotence Reconsidered

In Version 1, G is the only Absolute phenomenon: OG means G is totally omnipotent and thus causes even itself. But what if there exist other Absolute phenomena, such as logical, undeniable truths? This would mean Ax for some x other than G. By definition, Ax means that nothing different from x causes x – that is, G cannot cause x. In consideration of this possibility, we must redefine omnipotence accordingly:

<u>Def. Relative Omnipotence</u>

$$\forall x(Ox \Leftrightarrow \forall y(\neg Ay \Rightarrow x \blacktriangleright y))$$

This means x causes all *relative* (non-Absolute) phenomena. The term *omnipotence* and the predicate symbol O shall now mean *relative omnipotence* rather than *total omnipotence*.

Theory of Absolute Phenomena

Language (Undefined Terms)

Phenomenon, ►, and ⊏ are as in Version 1.

G is a particular phenomenon.

H is a particular phenomenon.

Definitions

Def. *A:* Absolute

$$\forall x(Ax \Leftrightarrow \neg\exists y(y \blacktriangleright x \wedge x \neq y))$$

x is not relative; that is, nothing different from *x* causes *x*.

Def. *C:* Causes Something Else

$$\forall x(Cx \Leftrightarrow \exists y\,(x \blacktriangleright y \wedge y \neq x))$$

x causes something other than itself.

Def. *O:* Relatively Omnipotent

$$\forall x(Ox \Leftrightarrow \forall y(\neg Ay \Rightarrow x \blacktriangleright y))$$

x causes all relative (non-Absolute) phenomena.

Axioms

From Version 1:

Note: A1 is not used here.

A2. Transitivity

$$\forall r \forall s \forall t \, ([r \blacktriangleright s \wedge s \blacktriangleright t] \Rightarrow r \blacktriangleright t)$$

A3. Non-Circularity

$$\forall r \forall s \, ([r \blacktriangleright s \wedge s \blacktriangleright r] \Rightarrow r = s)$$

Revised Version 1 Axioms:

A4. Complete Causation

$$\forall r \forall s \forall t \, ([r \blacktriangleright t \wedge s \sqsubset t \wedge \neg As \wedge r \neq s] \Rightarrow r \blacktriangleright s)$$

A5. Relative-Causal Extension

$$\forall z \, ((\neg Az \wedge Cz \wedge z \neq H) \Rightarrow z \sqsubset H)$$

New Axioms:

A6. Absolute Extension

$$\forall z \, (z \sqsubset G \Leftrightarrow (Az \wedge z \neq G))$$

A7. Transference

$$\forall r \forall s \forall t \, ([r \sqsubset s \wedge r \blacktriangleright t \wedge \neg t \sqsubset s \wedge t \neq s] \Rightarrow s \blacktriangleright t)$$

A8. Equipotency

$$\forall r \forall s \, ((Cr \wedge \forall x[(x \neq r \wedge x \neq s) \Rightarrow (r \blacktriangleright x \Leftrightarrow s \blacktriangleright x)]) \Rightarrow r = s)$$

A9. Existence of Causal Chains

$$\exists x \, (\neg Ax \wedge Cx)$$

A10. Partial Relativity

$$\forall r \forall s \, ((\neg Ar \wedge r \sqsubset s) \Rightarrow \neg As)$$

Justification for the New and Revised Axioms

Both A2 and A3 are exactly as in Version 1. Their justifications are in both Video 6 and Version 1 [2 and 3], as are the justifications for the original forms of A4 and A5.

Axiom 4 (Revised)

The analysis of A4 in Version 1 raises interesting questions about self-causing phenomena. The axiom says that whenever $r \blacktriangleright t \wedge s \sqsubset t$ it follows that $r \blacktriangleright s$. In other words, in order to cause a whole, one must cause each part of the whole.

If additionally Ss, then s might be both self-causing and relative, violating the Self-Causation Exclusion Principle. Is this an exception to A4? One possible solution is that this situation only occurs when $r = s$, preventing the unusual conclusion $Ss \wedge Rs$. If you believe that God is the only Absolute phenomenon, then this solution suffices: in fact it can be easily proven in Version 1 that God is the only Absolute phenomenon, and if the Self-Causation Exclusion Principle is used as an additional axiom, it follows trivially that God is the only self-causing phenomenon.

On the other hand, if you believe it is possible that multiple Absolute phenomena exist, then A4 must be revised to accommodate this situation. This is accomplished simply by adding an exception requiring $\neg As$: if a whole has parts, and something causes the whole, then it also causes each part, *except* for any parts which are Absolute.

One more issue to consider is what happens when one of the parts causes the whole. By causing the whole, the one part must also cause each part of the whole – including itself. This reasoning seems suspicious, so in order to avoid it, another exception is added: $r \neq s$.

Revised A4. Complete Causation

$$\forall r \forall s \forall t\ ([r \blacktriangleright t \wedge s \sqsubset t \wedge \neg As \wedge r \neq s] \Rightarrow r \blacktriangleright s)$$

Axiom 5 (Revised)

Similar revisions are made for A5. Also, the constant H is used in place of the original existential quantifier, and this simplifies the system of proofs that follow.

As now revised, A5 avoids the question of what it would mean, if it is even possible, for a phenomenon to be a part of itself. The revision also avoids the possibility (suggested in A4's revision above) where a phenomenon's parts include both relative and Absolute phenomena.

Revised A5. Causal Extension for Relative Phenomena

$$\forall z\ ((\neg Az \land Cz \land z \neq H) \Rightarrow z \sqsubset H)$$

In Version 1, an Absolute phenomenon is self-causing, which means it causes something and therefore is a part of a phenomenon analogous to *H* known to exist by the original A5. With this revision, A5 neither requires nor denies that any Absolute phenomenon be a part of *H*. It only requires certain *relative* phenomena to be a part of *H*: specifically, every relative phenomena, besides *H* itself, *which causes something else*.

As will be explained in a later section, the constant *H* could be removed entirely, and A5 generalized into an existential statement saying that something like *H* exists, as is the style in Version 1.

Axiom 6

A6. Absolute Extension

$$\forall z\ (z \sqsubset G \Leftrightarrow (Az \land z \neq G))$$

A6 concerns Absolute phenomena. It says that whatever Absolute phenomena exist (if any) can be taken together as parts of a whole, and this whole is the constant *G*. More precisely, A6 states that *every* Absolute phenomenon that is not equal to *G* is a part of *G*, and that *no* relative phenomenon is a part of *G*. A6 does not imply or even suggest that any Absolute phenomenon exists; if none exists, then *G* has no parts. Nor does A6 by itself imply that *G* itself is Absolute.

As will be explained in a later section, the constant *G* could be removed entirely, and A6 generalized into an existential statement saying that something like *G* exists, as is the style in Version 1.

A6 is justified on the same grounds as the justification given for A5 in Version 1. There, an analogy was made to the collective phenomenon of all the stars in the universe (called "All Stars"). It is intuitively obvious that the All Stars phenomenon exists. While it is less obvious that any Absolute phenomena exist, if any do, it is again intuitively obvious they can be gathered into a collective phenomenon, which A6 calls *G*.

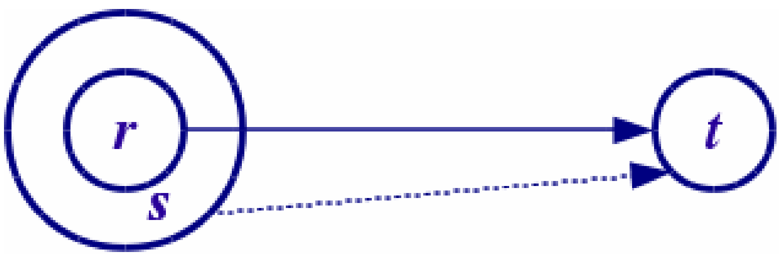
However, A6 contains a biconditional, whereas A5 contains only a conditional. This means that *G* contains *precisely* all Absolute phenomena that are not equal to *G* (*i.e. G* does *not* include any relative phenomenon), whereas, while *H* contains all relative phenomena that are not equal to *H* and that cause anything, *H* can *also* include Absolute phenomena, as well as relative phenomena that don't cause anything. This distinction is merely a question of how strictly the boundary is drawn. By the astronomical analogy, the All Stars phenomenon might, or might not, be permitted to include planets and other non-star things. The boundary for *H* is less strict than for *G*. Either way, the existence of these collective phenomena is intuitively obvious.

Axiom 7

A7. Transference

$$\forall r \forall s \forall t\ ([r \sqsubset s \wedge r \blacktriangleright t \wedge \neg t \sqsubset s \wedge t \neq s] \Rightarrow s \blacktriangleright t)$$

Consider the diagram below, showing three distinct phenomena *r*, *s* and *t*, where *r* is part of *s*, and *t* is outside *s*:



Axiom 7 says that if *r* causes *t*, then *s* also causes *t*. This *transfers* the potency of the part *r* to the whole *s*. In other words, whatever the parts cause, the whole also causes.

A7 is justified because *s* must necessarily have the ability to cause *t* since *s* includes *r* which causes *t*. For example, take the whole, *s*, to be the pianist Lang Lang and *r* to be his hands, which are a part of him. When he performs a Beethoven sonata, take *t* to be the experience that a member of the audience has of enjoying the music, which experience is outside of Lang Lang. A7 says that if Lang Lang's hands, *r*, which strike the keys on the piano causing the audience member to experience the music, then Lang Lang *himself*, *s*, also does so. This is intuitively obvious. Indeed, it will be Lang Lang himself, and not his hands, that the audience will applaud after the performance!

Axiom 8

A8. Equipotency

$$\forall r \forall s\ ((Cr \wedge\ \forall x[(x \neq r \wedge x \neq s) \Rightarrow\ (r \blacktriangleright x \Leftrightarrow s \blacktriangleright x)]) \Rightarrow r = s)$$

Consider the *effects* of a pair of phenomena on *other* phenomena. The two phenomena are e*quipotent* if (1) they cause something else (they have *some* effect) and (2) they cause exactly the same phenomena (their effects are *identical*). Axiom 8 states that equipotent phenomena are equal. Essentially this means that *effects determine uniqueness*: in order for 2 phenomena to be different, either they have no effect whatsoever, or there exists some effect which is caused by one but not by the other.

A8 states that, for any two phenomena r and s, if at least one of them (here r) causes something, *i.e.* Cr, and all *other* phenomena, x, that are distinct from both r and s, are either caused by both of them or by neither of them, then $r = s$. (Note: in sentential logic, the biconditional within A8 ($r \blacktriangleright x \Leftrightarrow s \blacktriangleright x$) is equivalent to the disjunction $(r \blacktriangleright x \wedge s \blacktriangleright x) \vee (\neg r \blacktriangleright x \wedge \neg s \blacktriangleright x)$, *i.e.* both r and s cause x or neither causes x.)

The very process by which science is conducted justifies A8, because science is based on *observation*, and hence on the *effects* that things have on the human senses, as well as on scientific instruments used to detect effects that the human senses cannot detect. For example, if we look at the apples hanging on a tree, no two apples will have the same effect on our senses, as they will each appear in a different *place*. Each apple reflects the sunlight from a different place, and the reflected light rays constitute *unique effects* of each apple. Even if two apples are aligned with our line of sight, one behind the other, and might be mistaken for a single apple, we can shift our position to a place where they will appear distinct, and this is precisely because the effects of the two apples differ. A8 says that when we always observe the same effects, no matter how we change our viewpoint and no matter what senses or instruments we use, then we are just looking at a *single* phenomenon. In other words, there is no scientific justification for looking at what appears *by all observations* to be a *single* apple, and declaring it instead to be a *pair* of distinct apples!

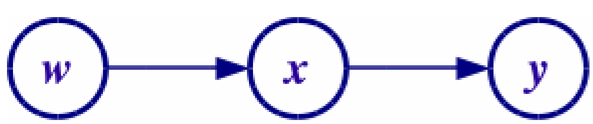
The inequality clauses $x \neq r \wedge x \neq s$ eliminate the absurd case where the single apple is claimed to be two apples, even though the external effects of both are identical, on the purported (but non-observable) grounds that one of them "causes" the other, thereby creating non-identical "effects."

<u>Axiom 9</u>

A9. Existence of Causal Chains

$$\exists x\, (\neg Ax \wedge Cx)$$

Axiom 9 states that there exists a phenomenon which both causes something else and is relative. In other words, at least one phenomenon, x, exists in the *middle* of a causal chain of three distinct phenomena:



$\neg Ax$ holds, because x is caused by w. Cx holds, because x causes y.

A9 is justified by plentiful examples in the real world, such as a set of three dominos, where the falling of the first, causes the falling of the second, which, in turn, causes the falling of the third. Unquestionably, at least one causal chain exists.

Axiom 10

A10. Partial Relativity

$$\forall r \forall s\ ((\neg Ar \land r \sqsubset s) \Rightarrow \neg As)$$

Axiom 10 states that whatever has a relative phenomenon as a part is also relative. Specifically, A10 says that if *r* is relative and is a part of *s*, then *s* is also relative.

A10 is justified because a relative phenomenon *r* depends on something else to cause it to exist, and hence the whole of *s* is also dependent, because *s* cannot exist unless each of its parts, including *r*, exists. For example, an automobile engine is a relative phenomenon that is caused by a manufacturing process. A car cannot exist until its engine has been manufactured. Thus, the car is also relative.

Theorems

The diagram shows which axioms and preceding theorems are used to prove each theorem.

T1. $\neg AH$

T2. $\exists x\ (x \blacktriangleright H \wedge\ Ax)$

T3. $\forall x\ [(\neg Ax \wedge x \neq G) \Rightarrow G \blacktriangleright x]$

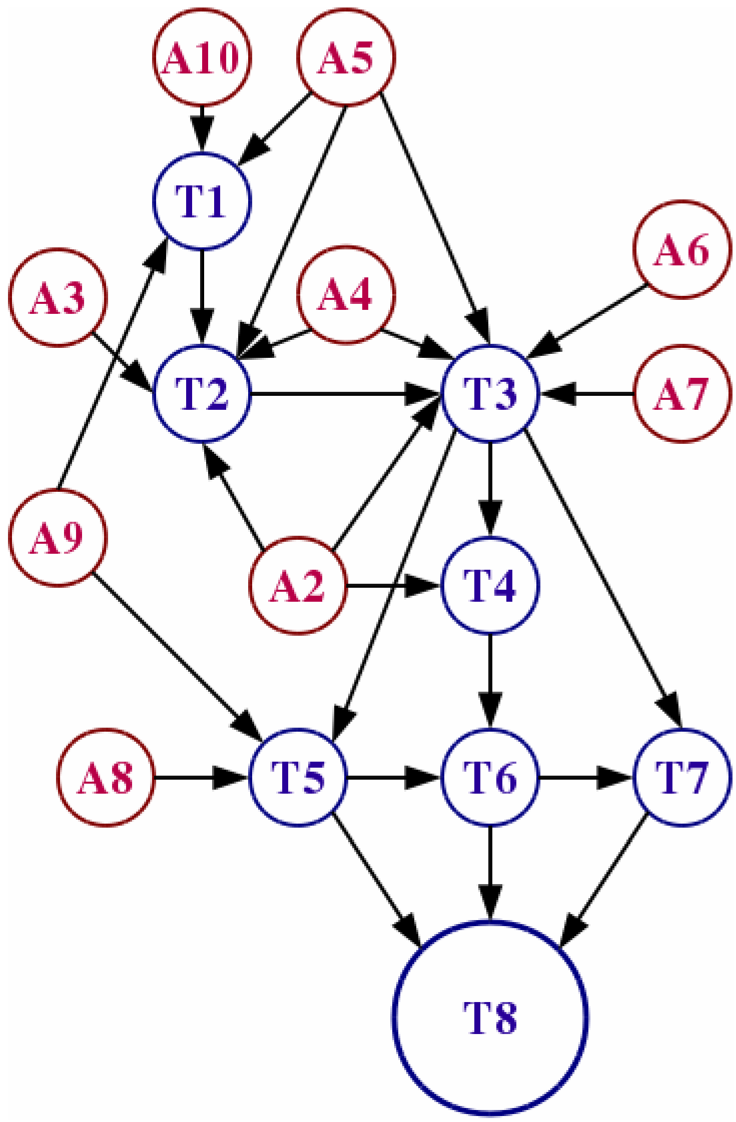
T4. $\forall z\ (z \blacktriangleright G \Rightarrow Oz)$

T5. $\forall z\ (Oz \Rightarrow z = G)$

T6. AG

T7. OG

T8. $\exists! g\ (Ag \wedge Og)$



Consequences of the Theorems

By T8, we know that the description

$$\iota g\,(Ag \wedge Og)$$

is proper. Since T6 and T7 prove *AG* and *OG*, it is clear that

$$G = \iota g\,(Ag \wedge Og).$$

<u>Definition of God</u>

God = *G*.

Thus, God is unique, omnipotent, and Absolute. God is the whole of Absolute phenomena. Since *G* is undefined, all of its properties are derived from A6, which says that *G* consists of exactly the Absolute phenomena. Thus, if there happen to exist any Absolute phenomena *other* than God, then, by A6, they must be *parts* of God—metaphorically, they are "cells" in the "body" of God.

Interestingly, mathematics appears to be Absolute. To understand the proofs presented herein requires *recognizing a part of God*.

Proofs of the Theorems

The proofs of T1-T8 are presented on the following pages.

Theorem T1. $\neg AH$

Ln	Statement	From	By
1	~~*Prove*~~ $\neg AH$	—	Claim
2	$\neg\neg AH$	—	AIP
3	$\exists x(\neg Ax \wedge Cx)$	A9	R
4	$\neg Aq \wedge Cq$	3	EI
5	$\neg Aq$	4	Simp
6	$q \neq H$	2, 5	LL
7	$\neg Aq \wedge Cq \wedge q \neq H$	4, 6	Adj
8	$(\neg Aq \wedge Cq \wedge q \neq H) \Longrightarrow q \sqsubset H$	A5	UI
9	$q \sqsubset H$	7, 8	MP
10	$\neg Aq \wedge q \sqsubset H$	5, 9	Adj
11	$(\neg Aq \wedge q \sqsubset H) \Longrightarrow \neg AH$	A10	UI, UI
12	$\neg AH$	10, 11	MP
13	*done*	2, 12	IP

Theorem T2. $\exists x(x \blacktriangleright H \wedge Ax)$

Ln	Statement	From	By
1	~~*Prove*~~ $\exists x(x \blacktriangleright H \wedge Ax)$	—	Claim
2	$\neg AH$	T1	R
3	$\neg\neg\exists y(y \blacktriangleright H \wedge H \neq y)$	2, Def A	UI, BC, MT
4	$e \blacktriangleright H \wedge H \neq e$	3	DN, EI
5	$e \blacktriangleright H$	4	Simp
6	$Ae \vee \neg Ae$	—	LEM
7	~~*Prove*~~ $Ae \Longrightarrow \exists x(x \blacktriangleright H \wedge Ax)$	—	Claim
8	Ae	—	ACP
9	$e \blacktriangleright H \wedge Ae$	5, 8	Adj
10	$\exists x(x \blacktriangleright H \wedge Ax)$	9	EG
11	*done*	10	CP
12	~~*Prove*~~ $\neg Ae \Longrightarrow \exists x(x \blacktriangleright H \wedge Ax)$	—	Claim
13	$\neg Ae$	—	ACP
14	$\neg\neg\exists y(y \blacktriangleright e \wedge e \neq y)$	13, Def A	UI, BC, MT
15	$f \blacktriangleright e \wedge e \neq f$	14	DN, EI
16	$\exists y(f \blacktriangleright y \wedge y \neq f)$	15	EG
17	Cf	16, Def C	UI, BC, MP
18	$f \blacktriangleright e$	15	Simp
19	$f \blacktriangleright e \wedge e \blacktriangleright H$	5, 18	Adj
20	$(f \blacktriangleright e \wedge e \blacktriangleright H) \Longrightarrow f \blacktriangleright H$	A2	UI, UI, UI
21	$f \blacktriangleright H$	19, 20	MP
22	$e \neq f$	15	Simp
23	~~*Prove*~~ $\neg e \blacktriangleright f$	—	Claim
24	$e \blacktriangleright f$	—	AIP
25	$e \blacktriangleright f \wedge f \blacktriangleright e$	18, 24	Adj
26	$e = f$	A3, 25	UI, UI, MP
27	*done*	22, 26	IP
28	$f \neq H$	5, 23	LL
29	~~*Prove*~~ Af	—	Claim
30	$\neg Af$	—	AIP
31	$\neg Af \wedge Cf \wedge f \neq H$	17, 28, 30	Adj
32	$(\neg Af \wedge Cf \wedge f \neq H) \Longrightarrow f \sqsubset H$	A5	UI
33	$f \sqsubset H$	31, 32	MP
34	$e \blacktriangleright H \wedge f \sqsubset H \wedge \neg Af \wedge e \neq f$	5,22,30,33	Adj
35	$(e \blacktriangleright H \wedge f \sqsubset H \wedge \neg Af \wedge e \neq f) \Longrightarrow e \blacktriangleright f$	A4	UI, UI, UI
36	$e \blacktriangleright f$	34, 35	MP
37	*done*	23, 36	IP
38	$f \blacktriangleright H \wedge Af$	21, 29	Adj
39	$\exists x(x \blacktriangleright H \wedge Ax)$	38	EG
40	*done*	39	CP
41	$\exists x(x \blacktriangleright H \wedge Ax)$	6, 7, 12	SC
42	*done*	41	DP

Theorem T3. $\forall x[(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x]$

Ln	Statement	From	By
1	~~*Prove*~~ $\forall x[(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x]$	—	Claim
2	~~*Prove*~~ $(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	—	Claim
3	$\neg Ax \wedge x \neq G$	—	ACP
4	$\neg Ax$	3	Simp
5	$x \neq G$	3	Simp
6	$x \sqsubset G \Longrightarrow (Ax \wedge x \neq G)$	A6	UI, BC
7	~~*Prove*~~ $\neg x \sqsubset G$	—	Claim
8	$x \sqsubset G$	—	AIP
9	Ax	6, 8	MP, Simp
10	*done*	4, 9	IP
11	$\neg\neg\exists y(y \blacktriangleright x \wedge x \neq y)$	4, Def A	UI, BC, MT
12	$j \blacktriangleright x \wedge x \neq j$	11	DN, EI
13	Cj	12, Def C	EG,UI,BC,MP
14	$j \blacktriangleright x$	12	Simp
15	~~*Prove*~~ $G \blacktriangleright x$	—	Claim
16	$\neg G \blacktriangleright x$	—	AIP
17	~~*Prove*~~ $\neg Aj$	—	Claim
18	Aj	—	AIP
19	$j \neq G$	14, 16	LL
20	$(Aj \wedge j \neq G) \Longrightarrow j \sqsubset G$	A6	UI, BC
21	$j \sqsubset G$	18, 19, 20	Adj, MP
22	$(j \sqsubset G \wedge j \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	A7	UI, UI, UI
23	$j \sqsubset G \wedge j \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G$	5, 7, 14, 21	Adj
24	$G \blacktriangleright x$	22, 23	MP
25	*done*	16, 24	IP
26	$z \blacktriangleright H \wedge Az$	T2	EI
27	$z \blacktriangleright H$	26	Simp
28	Az	26	Simp
29	~~*Prove*~~ $z \blacktriangleright j$	—	Claim
30	$\neg z \blacktriangleright j$	—	AIP
31	$j \neq H$	27, 30	LL
32	$z \neq j$	17, 28	LL
33	$\neg Aj \wedge Cj \wedge j \neq H$	13, 17, 31	Adj
34	$j \sqsubset H$	A5, 33	UI, MP
35	$(z \blacktriangleright H \wedge j \sqsubset H \wedge \neg Aj \wedge z \neq j) \Longrightarrow z \blacktriangleright j$	A4	UI, UI, UI
36	$z \blacktriangleright H \wedge j \sqsubset H \wedge \neg Aj \wedge z \neq j$	17,27,32,34	Adj
37	$z \blacktriangleright j$	35, 36	MP
38	*done*	30, 37	IP
39	$z \blacktriangleright x$	A2, 14, 29	Adj,UIx3,MP
40	$z \neq G$	16, 39	LL
41	$(Az \wedge z \neq G) \Longrightarrow z \sqsubset G$	A6	UI, BC
42	$z \sqsubset G$	28, 40, 41	Adj, MP
43	$(z \sqsubset G \wedge z \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	A7	UI, UI, UI
44	$z \sqsubset G \wedge z \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G$	5, 7, 39, 42	Adj
45	$G \blacktriangleright x$	43, 44	MP
46	*done*	16, 45	IP
47	*done*	15	CP
48	*done*	2	UP

Theorem T4. $\forall z(z \blacktriangleright G \Longrightarrow Oz)$

Ln	Statement	From	By
1	~~*Prove*~~ $\forall z(z \blacktriangleright G \Longrightarrow Oz)$	—	Claim
2	~~*Prove*~~ $z \blacktriangleright G \Longrightarrow Oz$	—	Claim
3	$z \blacktriangleright G$	—	ACP
4	~~*Prove*~~ $\forall y(\neg Ay \Longrightarrow z \blacktriangleright y)$	—	Claim
5	~~*Prove*~~ $\neg Ay \Longrightarrow z \blacktriangleright y$	—	Claim
6	$\neg Ay$	—	ACP
7	$(\neg Ay \wedge y \neq G) \Longrightarrow G \blacktriangleright y$	T3	UI
8	$(z \blacktriangleright G \wedge G \blacktriangleright y) \Longrightarrow z \blacktriangleright y$	A2	UI, UI, UI
9	~~*Prove*~~ $y \neq G \Longrightarrow z \blacktriangleright y$	—	Claim
10	$y \neq G$	—	ACP
11	$\neg Ay \wedge y \neq G$	6, 10	Adj
12	$G \blacktriangleright y$	7, 11	MP
13	$z \blacktriangleright G \wedge G \blacktriangleright y$	3, 12	Adj
14	$z \blacktriangleright y$	8, 13	MP
15	*done*	14	CP
16	~~*Prove*~~ $y = G \Longrightarrow z \blacktriangleright y$	—	Claim
17	$y = G$	—	ACP
18	$z \blacktriangleright y$	3, 17	SE
19	*done*	18	CP
20	$y = G \vee y \neq G$	—	LEM
21	$z \blacktriangleright y$	9, 16, 20	SC
22	*done*	21	CP
23	*done*	5	UP
24	Oz	4, Def O	UI, BC, MP
25	*done*	24	CP
26	*done*	2	UP

Theorem T5. $\forall z(Oz \Longrightarrow z = G)$

Ln	Statement	From	By
1	~~*Prove*~~ $\forall z(Oz \Longrightarrow z = G)$	—	Claim
2	~~*Prove*~~ $Oz \Longrightarrow z = G$	—	Claim
3	Oz	—	ACP
4	$\forall y(\neg Ay \Longrightarrow z \blacktriangleright y)$	3, Def O	UI,BC,MP
5	$\exists x(\neg Ax \wedge Cx)$	A9	R
6	$\neg Aq \wedge Cq$	5	EI
7	$\neg Aq$	6	Simp
8	Cq	6	Simp
9	$\neg Aq \Longrightarrow z \blacktriangleright q$	4	UI
10	$z \blacktriangleright q$	7, 9	MP
11	~~*Prove*~~ Cz	—	Claim
12	$\neg Cz$	—	AIP
13	$q \neq z$	8, 12	LL
14	$\exists y(z \blacktriangleright y \wedge y \neq z)$	10, 13	Adj, EG
15	Cz	14, Def C	UI,BC,MP
16	*done*	12, 15	IP
17	~~*Prove*~~ $\forall x[(x \neq z \wedge x \neq G) \Longrightarrow (z \blacktriangleright x \iff G \blacktriangleright x)]$	—	Claim
18	~~*Prove*~~ $(x \neq z \wedge x \neq G) \Longrightarrow (z \blacktriangleright x \iff G \blacktriangleright x)$	—	Claim
19	$x \neq z \wedge x \neq G$	—	ACP
20	$x \neq z$	19	Simp
21	$x \neq G$	19	Simp
22	~~*Prove*~~ $z \blacktriangleright x \Longrightarrow G \blacktriangleright x$	—	Claim
23	$z \blacktriangleright x$	—	ACP
24	$\neg\neg\exists y(y \blacktriangleright x \wedge x \neq y)$	20, 23	Adj,EG,DN
25	$\neg Ax$	24, Def A	UI,BC,MT
26	$\neg Ax \wedge x \neq G$	21, 25	Adj
27	$(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	T3	UI
28	$G \blacktriangleright x$	26, 27	MP
29	*done*	28	CP
30	~~*Prove*~~ $G \blacktriangleright x \Longrightarrow z \blacktriangleright x$	—	Claim
31	$G \blacktriangleright x$	—	ACP
32	$\neg\neg\exists y(y \blacktriangleright x \wedge x \neq y)$	21, 31	Adj,EG,DN
33	$\neg Ax$	32, Def A	UI,BC,MT
34	$\neg Ax \Longrightarrow z \blacktriangleright x$	4	UI
35	$z \blacktriangleright x$	33, 34	MP
36	*done*	35	CP
37	$z \blacktriangleright x \iff G \blacktriangleright x$	22, 30	CB
38	*done*	37	CP
39	*done*	18	UP
40	$Cz \wedge \forall x[(x \neq z \wedge x \neq G) \Longrightarrow (z \blacktriangleright x \iff G \blacktriangleright x)]$	11, 17	Adj
41	$z = G$	40, A8	UI,UI, MP
42	*done*	41	CP
43	*done*	2	UP

Theorem T6. AG

Ln	Statement	From	By
1	~~*Prove*~~ AG	—	Claim
2	$\neg AG$	—	AIP
3	$\exists y(y \blacktriangleright G \wedge G \neq y)$	Def A, 2	UI, BC, MT, DN
4	$z \blacktriangleright G \wedge G \neq z$	3	EI
5	$z \blacktriangleright G$	4	Simp
6	$G \neq z$	4	Simp
7	$z \blacktriangleright G \Longrightarrow Oz$	T4	UI
8	Oz	5, 7	MP
9	$Oz \Longrightarrow z = G$	T5	UI
10	$z = G$	8, 9	MP
11	*done*	6, 10	IP

Theorem T7. OG

Ln	Statement	From	By
1	~~*Prove*~~ OG	—	Claim
2	~~*Prove*~~ $\forall y(\neg Ay \Longrightarrow G \blacktriangleright y)$	—	Claim
3	~~*Prove*~~ $\neg Ay \Longrightarrow G \blacktriangleright y$	—	Claim
4	$\neg Ay$	—	ACP
5	AG	T6	R
6	$y \neq G$	4, 5	LL
7	$(\neg Ay \wedge y \neq G) \Longrightarrow G \blacktriangleright y$	T3	UI
8	$\neg Ay \wedge y \neq G$	4, 6	Adj
9	$G \blacktriangleright y$	7, 8	MP
10	*done*	9	CP
11	*done*	3	UP
12	OG	Def O, 2	UI, BC, MP
13	*done*	12	DP

Theorem T8. $\exists! g(Ag \wedge Og)$

Ln	Statement	From	By
1	~~*Prove*~~ $\exists! g(Ag \wedge Og)$	—	Claim
2	~~*Prove*~~ $\forall x((Ax \wedge Ox) \iff x = G)$	—	Claim
3	~~*Prove*~~ $(Ax \wedge Ox) \implies x = G$	—	Claim
4	$Ax \wedge Ox$	—	ACP
5	Ox	4	Simp
6	$Ox \implies x = G$	T5	UI
7	$x = G$	5, 6	MP
8	*done*	7	CP
9	~~*Prove*~~ $x = G \implies (Ax \wedge Ox)$	—	Claim
10	$x = G$	—	ACP
11	$AG \wedge OG$	T6, T7	Adj
12	$Ax \wedge Ox$	10, 11	SE
13	*done*	12	CP
14	$(Ax \wedge Ox) \iff x = G$	3, 9	CB
15	*done*	14	UP
16	$\exists y \forall x((Ax \wedge Ox) \iff x = y)$	2	EG
17	$\exists! g(Ag \wedge Og)$	16	MUFS
18	*done*	17	DP

What If Constants G and H Are Excluded from the Theory?

In Version 1, no constant H is needed. This is because one of the axioms is an existential statement:

Version 1 A5: $\exists r \forall z\, (Cz \Rightarrow z \sqsubset r)$.

Instead of naming a constant (like H), the axiom simply states that such a thing exists.

In TAP, we can similarly exclude both constants G and H by generalizing 2 of the axioms into existential statements:

$\exists h \forall z\, ((\neg Az \wedge Cz \wedge z \neq h) \Rightarrow z \sqsubset h)$. (A5, EG)

$\exists g \forall z\, (z \sqsubset g \Leftrightarrow (Az \wedge z \neq g))$. (A6, EG)

How does this affect the theorems mentioning G and H? It won't work just to use EG on them. This would change T1, which states $\neg AH$, to become

$\exists y\, (\neg Ay)$

which has no logical connection to the h in the generalized A5. Instead, consider what it means to be either G-like or H-like. This is easily elucidated in 2 steps. For each of these constants:

1. In the axiom mentioning the constant, replace the constant with a new variable.

2. Create a new definition generalizing the new variable.

Following these steps for A5 and for A6, we get:

Def J: $\forall g(Jg \Leftrightarrow \forall z\, (z \sqsubset g \Leftrightarrow (Az \wedge z \neq g)))$.

Def K: $\forall h(Kh \Leftrightarrow \forall z\, ((\neg Az \wedge Cz \wedge z \neq h) \Rightarrow z \sqsubset h))$.

Clearly, J just means to be G-like, and K just means to be H-like, so we can abbreviate A5 and A6 as KH and JG. It is now straightforward to modify the entire theory by eliminating every mention of G and H. In their place we use generalized statements about G-like and H-like phenomena. The simplest modifications are of A5 and A6:

Modified A5: $\exists h\, (Kh)$.

Modified A6: $\exists g\, (Jg)$.

Each theorem $T(G)$ mentioning G becomes

$$\forall g\ (Jg \Rightarrow T(g)).$$

Similarly, each theorem $T(H)$ mentioning H becomes

$$\forall h\ (Kh \Rightarrow T(h)).$$

For example, consider T1: $\neg AH$. This becomes

Modified T1: $\forall h\ (Kh \Rightarrow \neg Ah)$.

In order to prove the modified T1, assume Kh where h is arbitrary. Because of this assumption, A5 is not needed as Kh amounts to the same thing. But in other proofs, whenever an H-like phenomenon is desired, Modified A5 says that one exists, and then the Modified T1 and T2 each say the existing one has the relevant property.

These same principles hold for theorems T3-T7 which concern G. The final statement T8 remains the same because it does not explicitly mention either G or H. But its proof must be altered slightly as outlined in the preceding paragraph: by Modified A6, something G-like exists (call it g); Modified T6 and T7 together say that anything G-like is Absolute and omnipotent, hence g must be; and by Modified T5, again since g is G-like, anything which is omnipotent is equal to g, establishing uniqueness.

Revisiting the Principle of Sufficient Reason

Tortoise: Good. And now I have yes-answer ω +1. Armed with it, I can proceed to accept the hypothesis that today is your birthday, if it is valid to do so. Would you be so kind as to counsel me on that matter, Achilles?

Achilles: What is this? I thought I had seen through your infinite plot. Now doesn't yes-answer ω +1 satisfy you? All right. I'll give you not only yes-answer ω +2, but also yes-answers ω +3, ω +4, and so on.

— *Birthday Cantatatata...*, Douglas Hofstadter [5 at p. 463]

The Principle of Sufficient Reason is used in Version 1 in the form of A1. It is justified because all Absolute phenomena are assumed to be self-causing. Since TAP allows Absolute phenomena to be entirely causeless, the original A1 cannot be assumed.

A new interpretation of Sufficient Reason, the revised A1 below, will now be used in conjunction with some of, but not all, the other axioms to prove T8. Technically this forms a different theory, since it uses different axioms, but it is only a minor variation. More precisely, it is a *subtheory* of TAP, meaning that any statement which can be proven in the variation can also be proven from TAP.

Revised A1: Principle of Sufficient Reason

$$\forall x\ (\neg Ax \Rightarrow \exists y\ (Ay \wedge y \blacktriangleright x))$$

This says that, for every *relative* phenomenon, x, there exists an *Absolute* phenomenon, y, that causes it — *i.e.* that gives x reason to exist.

An Absolute phenomenon is already self-sufficient, somehow supplying its own reason to exist. All other phenomena are relative, requiring a sufficient reason. If the reason given is also relative, then a deeper reason is required. Might this process continue indefinitely? While many philosophers and theologians have posited that an infinite regress of causes is impossible, skeptics will refute this by suggesting that the universe could be infinite.

Considering that there is no evidence that the universe is finite, and indeed it may be impossible to determine scientifically the truth of the matter, this axiom *allows* the possibility of infinite regression. It states that, finite or not, the *entire regress of causes* which leads to a relative phenomenon *must have a sufficient reason to exist.* Such a sufficient reason must be Absolute. In order to express this in more precise terms, some sophisticated mathematics is needed.

Justification for Revised A1, Part 1: Sequences or Chains

Given any *causal chain*, allow

A-B-C-D, etc.

to abbreviate

A ▶ B ∧ B ▶ C ∧ C ▶ D, etc.

Each of A, B, C, and D is called a *link* in the chain.

Directly from A2 and A3, it is obvious that in any causal chain:

1. Each link causes every link to the *right* of it, and

2. If a phenomenon occurs twice in one chain, then (a) it is *self-causing*, and (b) it is *equal to each link in between* its two occurrences. For example, if

   A-B-C-D-E-F-G-D-H-I-J-K

   then D is self-causing, and by A3, D = E = F = G.

Consequently, if none of the links in a particular chain is self-causing, then no phenomenon may occur twice in that chain. If the Self-Causation Exclusion Principle is accepted, which requires that any relative phenomenon is not self-causing, then no relative phenomenon can occur twice in one chain. However, for purposes of this justification, SCEP need not be assumed, because chains are always constructed to ensure that no phenomenon occurs more than once.

The Consequences of Denying Revised A1

Consider an arbitrary relative phenomenon, *x*. Is there an Absolute phenomenon which causes *x*? A1 says yes, but a skeptic may have doubts. Suppose that A1 is false.

Anti-A1 Assumption:

$$\neg\exists y\ (Ay \land y \blacktriangleright x)$$

Since *x* is relative, some x_1 exists which causes *x* and *by the assumption, anything which causes x must be relative*, including x_1. By induction, there must exist an infinite chain:

$$C_1 = \ldots x_{100}\text{-}x_{99}\text{-}x_{98}\ldots x_3\text{-}x_2\text{-}x_1$$

such that no phenomenon appears twice in the chain. First, x_1 causes x and $x \neq x_1$. Next consider an arbitrary x_n where every link to the right of it causes x. Clearly, x_n also causes x, which means, by our assumption, that x_n is also relative, and therefore some x_{n+1} must precede it so that x_{n+1} causes x_n, and x_{n+1} does not occur anywhere else in the chain (if it did occur elsewhere in the chain, it would create a loop, violating A3). This of course must go on infinitely, otherwise it would have to terminate on an Absolute phenomenon that causes x, contrary to the assumption.

A strict finitist who believes that infinity is an impossibility which is never actually realized in the universe would have to reject the infinitude of C_1 and instead accept A1.

Dismissing this extreme finitist objection and continuing with the assumption that no Absolute phenomenon causes x, C_1 must exist, and it explains the existence of phenomenon x via an infinite causal chain. A simple, intuitive convention suggests that a causal chain is itself a phenomenon, where each link *is a part of* the chain:

$$x_1 \sqsubset C_1, x_2 \sqsubset C_1, x_3 \sqsubset C_1, \ldots$$

Because each of x_1, x_2, x_3, etc. cause x, A7 applies (since x itself is not a link in chain C_1) revealing that *the whole*, C_1, *also* causes x. Hence, by our assumption, C_1 is also relative. Moreover, another infinite chain C_2 must exist which likewise causes C_1. Now it should be clear that infinite chains C_2, C_3, C_4, C_5, etc. must all exist, and in particular:

$$D_1 = \ldots C_{100} \text{-} C_{99} \text{-} C_{98} \ldots C_3 \text{-} C_2 \text{-} C_1$$

must exist. Of course the same reasoning applies to D_1, *and so forth*. Imagine, all this from just an arbitrary relative x! In fact this process is so remarkable that it deserves a name: call it the Transfinite Causation of x (TCx). Indeed, TCx is *itself* a phenomenon. Now, consider the following question:

<u>Is TCx Absolute?</u>

Certainly, one of the definitive characteristics of TCx is that it causes x! But by assumption, anything that causes x is *relative*, and it seems no progress has been made whatsoever: something exists which causes TCx, and another infinite chain exists, and— but wait! This process was already elucidated: it was named the Transfinite Causation of x, or TCx!

It seems as though TCx causes TCx! As will be explained in greater detail below, TCx is *self-causing*, which contradicts SCEP because, by assumption, TCx is also *relative*!

Justification for Revised A1, Part 2: Set Theory and Cantor’s Transfinite Ordinals

The previous section’s demonstration of infinite causal chains echoes Georg Cantor’s construction of the Transfinite Ordinals, which was a substantial contribution to the foundation of all mathematics.

In Set Theory, there are only two undefined terms: “set” and ∈ which means “is an element of.”

A = {B, C, D}

means A is the set whose elements are precisely B, C, and D:

B ∈ A, C ∈ A, and D ∈ A, and nothing else is an element of A.

All of the objects within Set Theory are sets, and no two sets have exactly the same elements, so {B, C, D} = {C, D, B} because they have exactly the same elements — the order of the elements is irrelevant. Although the elements of a set can be written in any order, sometimes it is more expressive or informative to use a particular order. Any such ordering is used exclusively for readability and has no logical consequences.

The most basic set is called the empty set, or the null set: predictably, it has no elements, and since no two sets have exactly the same elements, the empty set is unique. It is written like this:

{ } = the empty set.

Since nothing is inside the curly brackets, nothing is an element of the empty set. Embodying the concept of “nothingness” within formal Set Theory, the empty set is used as the definition for the familiar concept of 0:

0 = { }.

Since all of the objects in formal Set Theory are sets, in order for a concept to be *accessible* within the theory, it must be *defined in terms of sets*. The empty set is the first (and smallest) Ordinal number, and it is used as the single basic building block for constructing all of the Ordinals (somewhat paradoxically, it appears that “everything” comes out of “nothing”, so to speak). The next Ordinal has exactly one element: the set just defined, 0. In Set Theory, this set is called 1:

1 = {0}.

Notice that the set 0 has no elements, and the set 1 has a single element, which is 0. The set 2 has 2 elements, the set 3 has 3 elements, and so forth.

2 = {0,1}

$3 = \{0,1,2\}$

...

$n+1 = \{0,1,...,n\}$

This schema comprises what is usually called the Natural numbers (aka the non-negative integers), all defined in terms of sets.

Within Any Ordinal, ∈ Determines a Linear Ordering

Given any 2 Natural numbers m and n (as defined in the above schema), it is easy to see that whenever $m < n$:

1. $m \in n$, and

2. every element of m is also an element of n: this is called the ⊆ relationship, and it is written $m \subseteq n$, or m is a *subset* of n.

More generally, for any 2 Natural numbers m and n, one of three possibilities must hold:

$$m \in n \vee n \in m \vee m = n.$$

This trichotomy corresponds to the linear ordering trichotomy of the < relation:

$$m < n \vee n < m \vee m = n.$$

Indeed, the term "Ordinal" represents the intuitive concept of linear ordering, and the ∈ relation determines every Ordinal's ordering.

Transcending the Natural Numbers

As soon as the Natural numbers are understood, the concept of infinity is realized: obviously there is no end to the series. But in the case of causal chains, it is not entirely obvious whether or not there exists an end to any chain (that is, whether Absolute phenomena exist ending the chains). However, several figures throughout history have voiced their opinions on the subject. More than 2000 years ago, Aristotle wrote that an infinite regress of causes is strictly impossible, and therefore a First Cause must exist. But in more recent centuries, philosophers have questioned this assumption, suggesting that an infinite regress is perfectly acceptable, and therefore no First Cause necessarily exists.

However, as Cantor demonstrated, even an infinite regress is not the end of the story. Just as infinite chains of infinite chains are illustrated above with D_1, the Ordinal numbers continue past the Natural numbers, starting with the set of all Natural numbers itself, which in Set Theory is usually called ω:

$$\omega = \{0, 1, 2, 3, \dots\}.$$

This Ordinal, like 0, has no immediate predecessor. It is called a limit Ordinal, and it is the first and smallest one.

$$\omega+1 = \{0, 1, 2, 3, ..., \omega\}.$$

This Ordinal ends at ω, so it is not a limit Ordinal. It includes an entire infinite series, and its immediate predecessor is ω.

$$\omega+2 = \{0, 1, 2, 3, ..., \omega, \omega+1\}$$

...

$$2\omega = \{0, 1, ..., \omega, \omega+1, \omega+2, \dots\}.$$

This is the second limit Ordinal.

...

It is easy to see that $\in$ continues to order the Ordinals, and this is in fact their definitive characteristic. *Informally and recursively defined, any Ordinal is equal to the set of all Ordinals preceding it.* Since nothing precedes the empty set 0, it indeed equals the set of all Ordinals preceding it, and so it is the first Ordinal. Next, 1 equals the set of all Ordinals preceding it (only 0 precedes it), and so forth, all the way to ω and its successors.

The Paradoxes of Naïve Set Theory

When Set Theory was first formalized, the term "set" or "class" originally meant a collection of objects which all share a common property: for any property Fx, the extension of F is the class of all objects x such that Fx. In other words,

$$x \in F \Leftrightarrow Fx.$$

F is called a *class-predicate*, because it is used as both a class and as a predicate. But what if some property, Fx, is a property *about classes*? For example suppose Px means "x is a class." Then P is the class of all classes. But since PP, P has the peculiar property of being its own element. Call this property Qx :

$Qx \Leftrightarrow x \in x.$

Since $P \in P$, QP. Equivalently, $P \in Q$. (Also, since Q is a class, PQ, which means $Q \in P$.)

Just as the class of all odd integers is itself *not* an odd integer, it is normal for a class *not* to be its own element. Call this opposite property Rx:

$Rx \Leftrightarrow \neg x \in x.$

The Law of Excluded Middle says that any class is either an element of itself, or it is not:

$x \in x \vee \neg x \in x.$

In other words:

$Qx \vee Rx.$

How about Q? Is QQ true, or is it RQ? Oddly enough, either one could be true, but certainly not both at the same time. How about R? This question presents a serious problem—in fact, a *fatal* problem. Is the class of all classes *not* containing themselves a class which *does* contain itself?

Suppose QR. The definition of Q says this means R is in its own class:

$R \in R$, and equivalently RR. But the definition of the predicate R says the opposite:

$\neg R \in R$. This is a contradiction.

Therefore, the supposition QR was false, so $\neg QR$ must be true, meaning R is *not* in its own class:

$\neg R \in R$, and equivalently $\neg RR$, which means:

$\neg\neg R \in R$, another contradiction.

This is called Russell's Paradox, discovered by Bertrand Russell. It illustrates that the general method of allowing a class to play both roles simultaneously — as predicate, and also as a predicated object — is inconsistent (*i.e.* it produces contradictions), unless special rules are followed to avoid any inconsistency (such as Russell and Whitehead carefully do in *Principia Mathematica* [7]). Since the method without any special rules is inconsistent, it is now called *naïve* set theory. Still, it is often useful to refer to the group of all sets (or some other kind of object) which share some property, and the term "class" is commonly used for this purpose. But formally, a *proper class* is not a set. While an *object* of Set Theory cannot be any of these proper classes, a *formula* of Set Theory can implicitly express one. For example the formula $\neg\, x \in x$, where x is a free variable, implicitly expresses Russell's anomaly R. But since R is a proper class, it cannot be the value of the variable x, and the paradox above asking whether $R \in R$ is formally not a grammatical sentence of Set Theory.

This concept of having separate *types* for "set" and "class" is related to the Theory of Types presented in *Principia Mathematica,* which resolves the paradox in a similar way. In the Theory of Types, every object must be of some type. An object of a higher type can only have as its elements objects of a lower type. Since "class" is the next higher type from "set", the elements of a class are always sets, so there can be no class of classes (instead, a higher type can be created, "meta-class", whose elements would be classes). But since "set" is the lowest-level, baseline type (in most versions of Set Theory), the elements of a set are in fact other sets, where the sets all have to be formed according to certain rules, following from the axioms. These rules circumvent all of the paradoxes. For example, the Axiom of Subsets allows the formation of a set based on any class-predicate F, but it requires that the newly formed set *be a subset of some already existing set*. In other words, while the class-predicate F is not itself a set, it can be intersected with any existing set to form a new set. The *intersection* $A \cap B$ is the set containing everything which is both an element of A and an element of B. If A and B have no element in common, then their intersection is the empty set.

Another interesting dilemma is the Burali-Forti Paradox. Consider the property of being an Ordinal and the corresponding class of all Ordinals, called Ω. Can Ω be a set? Suppose it is.

As the informal definition above states, an Ordinal is equal to the set of all Ordinals preceding it. Since 0 is an Ordinal, $0 \in \Omega$; likewise for 1, 2, 3, and the rest of the Naturals, and ω, and so forth: Ω consists of all of the Ordinals. Thus by the $\in$-ordering, all Ordinals precede Ω — that is, Ω is equal to the set of all Ordinals *preceding* it — but this means that Ω is *itself* an Ordinal! In other words:

$\Omega \in \Omega$

But this means Ω precedes itself! This defies the notion of a linear ordering; it is impossible to create an order in which some object precedes itself. Thus, Ω cannot be a true Ordinal. But so long as Ω is *not* an Ordinal:

$\neg\, \Omega \in \Omega$, which means it satisfies the definition of Ordinal.

This contradiction is called the Burali-Forti Paradox, and it is closely related to Russell's Paradox.

Ω Represents Order

On the one hand, there is no greatest Ordinal, just as there is no greatest Natural number. But on the other hand, Ω is just like an Ordinal, and all Ordinals precede it. Since it cannot be a set without contradicting itself, it must be of a different *type*: it is a *class*, and since the elements of a class can only be sets, it must exclude itself. What, then, does Ω represent? It represents *order itself*. Anywhere that some ordering occurs in the world, a tiny, infinitesimal piece of Ω represents this. For example, TC*x* as illustrated above is a linear ordering. What is the structure of this ordering? Is it the whole of Ω? Or is it just a piece of it, *i.e.* a particular Ordinal?

Suppose some Ordinal represents the structure of TC*x*. If TC*x* is relative, then there is a larger Ordinal which structures yet another chain. Continuing with the assumption that there is no Absolute cause of *x*, clearly the *only* structure which can represent the entire causal chain for *x* is Ω.

But this requires an Absolute phenomenon! Why? Since the whole of Ω cannot be surpassed, no Ordinal exists which can represent a chain with a phenomenon preceding (and thereby causing) the entire series x_Ω.

In other words, in positing that every cause of *x* is *relative*, there is actually a *hidden assumption* that the *entire infinite process* which somehow ultimately arrives at *x* is really an *Absolute process*.

All of this means that the Anti-A1 Assumption (*i.e.* that our revised A1 is false), has *no* justification and in fact leads to a contradictory result. A1 as revised here in Version 2 is therefore entirely justified.

Indeed, according to Cantor, who discovered Ω, God is that same Absolute Infinite captured by Ω. In his own words:

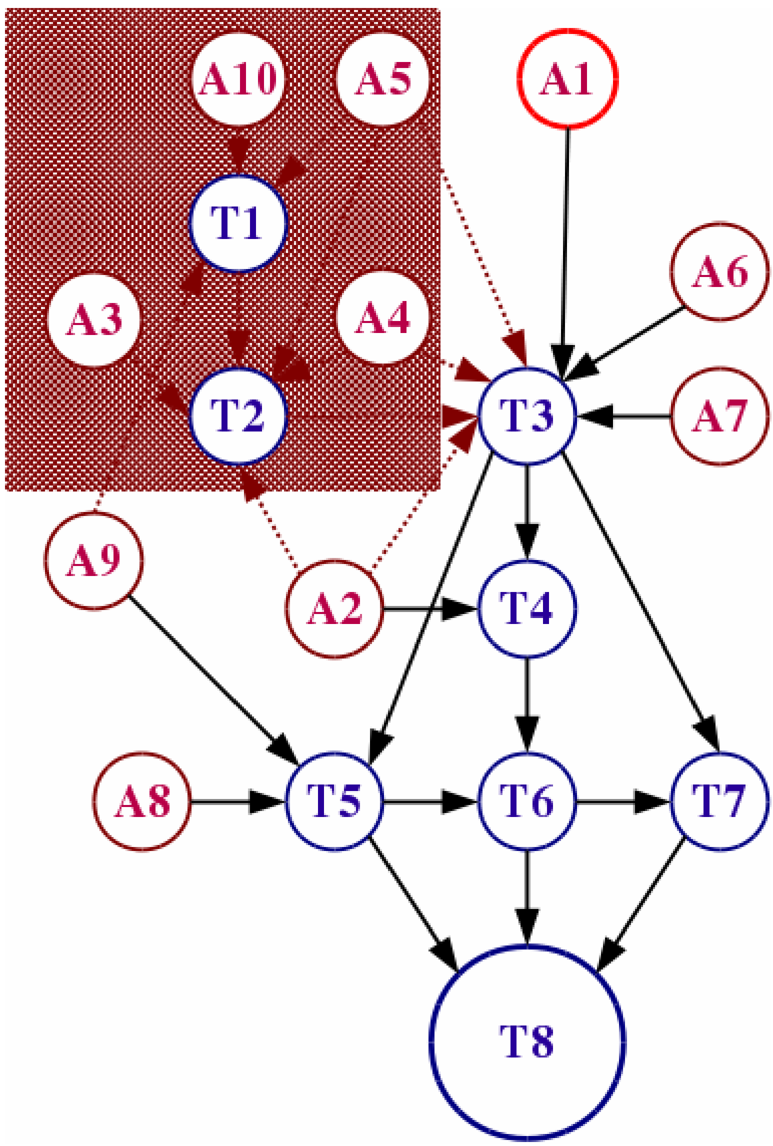
> "The actual infinite arises in three contexts: first when it is realized in the most complete form, in a fully independent otherworldly being, *in Deo*, where I call it the Absolute Infinite or simply Absolute; second when it occurs in the contingent, created world; third when the mind grasps it *in abstracto* as a mathematical magnitude, number or order type." [1 at p. 9 of Rucker]

Proving T8 from A1

If A1 is fully understood and accepted, then the proof of T8 is much simpler. The diagram below illustrates that by using Revised A1, the axioms and theorems in the boxed area are no longer needed to prove T3-T8.

The solid black arrows show that axioms A1, A6, and A7 are sufficient to prove T3, as will be demonstrated in the alternate proof of T3 on the next page. In the original proof, T1, T2, A3, A4, A5, and A10 were all used, but they are no longer necessary, so their arrows to T3 are shown in red. A2 is still needed to prove T4, and A8 and A9 are still needed for T5.

This means that even if all of the statements in the boxed area are rejected (except for A3, which is so essential that its falsity would render the "cause" relation meaningless and also provide a counter-example falsifying A8), the rest of the theorems still follow from the rest of the axioms.



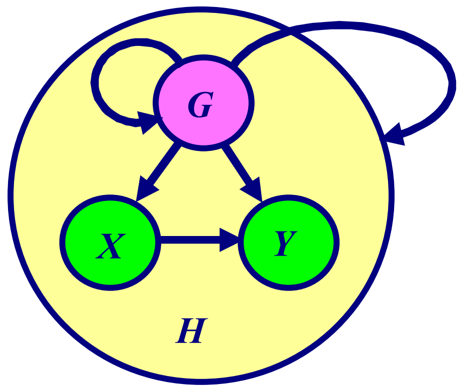
Alternative Proof of Theorem T3. $\forall x[(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x]$

Ln	Statement	From	By
1	~~*Prove*~~ $\forall x[(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x]$	—	Claim
2	~~*Prove*~~ $(\neg Ax \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	—	Claim
3	$\neg Ax \wedge x \neq G$	—	ACP
4	$\neg Ax$	3	Simp
5	$x \neq G$	3	Simp
6	$x \sqsubset G \Longrightarrow (Ax \wedge x \neq G)$	A6	UI, BC
7	~~*Prove*~~ $\neg x \sqsubset G$	—	Claim
8	$x \sqsubset G$	—	AIP
9	Ax	6, 8	MP, Simp
10	*done*	4, 9	IP
11	$\neg Ax \Longrightarrow \exists y(Ay \wedge y \blacktriangleright x)$	A1	UI
12	$\exists y(Ay \wedge y \blacktriangleright x)$	4, 11	MP
13	$Aj \wedge j \blacktriangleright x$	12	EI
14	Aj	13	Simp
15	$j \blacktriangleright x$	13	Simp
16	$j = G \vee j \neq G$	—	LEM
17	~~*Prove*~~ $j \neq G \Longrightarrow G \blacktriangleright x$	—	Claim
18	$j \neq G$	—	ACP
19	$Aj \wedge j \neq G$	14, 18	Adj
20	$(Aj \wedge j \neq G) \Longrightarrow j \sqsubset G$	A6	UI, BC
21	$j \sqsubset G$	19, 20	MP
22	$(j \sqsubset G \wedge j \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G) \Longrightarrow G \blacktriangleright x$	A7	UI, UI, UI
23	$j \sqsubset G \wedge j \blacktriangleright x \wedge \neg x \sqsubset G \wedge x \neq G$	5, 7, 15, 21	Adj
24	$G \blacktriangleright x$	22, 23	MP
25	*done*	24	CP
26	~~*Prove*~~ $j = G \Longrightarrow G \blacktriangleright x$	—	Claim
27	$j = G$	—	ACP
28	$G \blacktriangleright x$	15, 27	SE
29	*done*	28	CP
30	$G \blacktriangleright x$	16, 17, 26	SC
31	*done*	30	CP
32	*done*	2	UP

Models and Consistency

Here is a simple model which satisfies the 5 axioms from Version 1, the 5 new axioms A6-A10, and the 3 revised axioms, including A1. Its existence proves that both theories are individually consistent, and also that they are consistent with each other.

A Simple Model



Phenomena:

G, H, X, Y

Parts:

Everything is a part of *H* (except *H* is not a part of itself).

G, *X* and *Y* have no parts.

Causes:

G causes everything, including itself.

X ► *Y*.

Neither *Y* nor *H* causes anything.

It is easy to verify that this model satisfies all the axioms:

A1 & Revised A1: everything is caused by *G*, which is Absolute – satisfied.

A2: the only causal chain with 3 distinct links is *G*-*X*-*Y*, and *G* ► *Y* – satisfied.

A3: Clearly there are no 2-way causal loops – satisfied.

A4 & Revised A4: Only *H* has any parts. *G* causes *H* and all of its parts – satisfied.

A5 & Revised A5: Only *G* and *X* cause anything, and both are part of *H* – satisfied.

A6: *G* has no parts because nothing else is Absolute – satisfied.

A7: Only *H* has parts, but since nothing is outside of *H*, A7 does not apply – satisfied.

A8: *G* is different from *X* because *G* causes *H* and *X* does not. A8 does not apply for phenomena that do not cause anything, and nothing else causes anything – satisfied.

A9: $G \blacktriangleright X$ and $X \blacktriangleright Y$ so, $\neg AX \wedge CX$ – satisfied.

A10: Only *H* has parts, and $\neg AH$ bceause $G \blacktriangleright H$ – satisfied.

Significantly, if you remove the self-causing loop from *G*, so that *LG* instead of *SG*, then all axioms of TAP are still satisfied (while Version 1's A1 and A4 are not). If any number of Absolute phenomena are added to the model, then they must each be a part of *G*, and they likewise have the option to be with or without self-causing loops.

Causal Equivalence Principle

If models *M* and *N* are exactly the same except that some Absolute phenomena are self-causing in one model but causeless in the other (as suggested with *G* above being either with or without the self-causation loop), then *M* and *N* are *causally equivalent*.

From this definition, the Causal Equivalence Principle states that in TAP, *there is no meaningful difference between a self-causing Absolute phenomenon and a causeless Absolute phenomenon*:

Causal Equivalence Principle:

Given any pair of models *M* and *N*,

M and *N* are causally equivalent ⇒ (*M* satisfies TAP ⇔ *N* satisfies TAP).

Even without having established the formal metamathematics to prove this Principle, it is straightforward to prove it *informally*. Suppose that *M* and *N* are causally equivalent. This means they are identical with respect to: all *relative* phenomena, all *part* relationships, and all *cause* relationships *other than self-causation*. Assuming that *M* and *N* are not equal, there must exist *m* in *M* and *n* in *N* which are both Absolute, but either $Sm \wedge Ln$, or $Sn \wedge Lm$. Assume the former: $Sm \wedge Ln$ (obviously it would make no difference if $Sn \wedge Lm$ were assumed instead).

Consider that instead of TAP, the Causal Equivalence Principle can hold for other theories or even individual axioms. First, it will be shown that the Principle holds independently for all of the axioms:

M satisfies $A_i \Leftrightarrow N$ satisfies A_i

except A4.

A2. To make a model fail to satisfy A2,

$$[r \blacktriangleright s \wedge s \blacktriangleright t] \Rightarrow r \blacktriangleright t$$

must fail for some r, s and t. This is impossible if either s or t is Absolute, because that would mean either $r = s$ or $s = t$, and $r \blacktriangleright t$ follows immediately from either of these. Hence s and t must be *relative*. This means s and t behave exactly in the same way in both M and N, including with respect to whether r causes either or both of them. Accordingly, if either model fails to satisfy A2, then so does the other model.

A3. To make a model fail to satisfy A3,

$$[r \blacktriangleright s \wedge s \blacktriangleright r] \Rightarrow r = s$$

must fail for some r and s. This would mean by definition that both r and s are relative. This means r and s behave exactly in the same way in both M and N, including with respect to whether each causes the other. Accordingly, if either model fails to satisfy A3, then so does the other model.

A5. This only applies to *relative* phenomena, so it bears no consequences on causal equivalence.

$$\forall z\, ((\neg Az \wedge Cz \wedge z \neq H) \Rightarrow z \sqsubset H)$$

There is no room for m or n to have any effect on whether A5 is satified. Accordingly, if either model satisfies A5, then so does the other model.

A6. Similarly, A6 applies universally to all Absolute phenomena, including both m and n.

$$\forall z\, (z \sqsubset G \Leftrightarrow (Az \wedge z \neq G))$$

Both m and n are in G, or are both equal to G, regardless of whether they are self-causing or causeless.

A7. To make a model fail to satisfy A7,

$$\forall r \forall s \forall t\, ([r \sqsubset s \wedge r \blacktriangleright t \wedge \neg t \sqsubset s \wedge t \neq s] \Rightarrow s \blacktriangleright t)$$

must fail for some r, s and t. Since M and N are causally equivalent, the "part of" relationships are identical in both, as are the "causes" relationships with respect to any pair of distinct phenomena. However, as to the clause $r \blacktriangleright t$, when m is substituted in for both, the result $m \blacktriangleright m$ is true, whereas with n, the result $n \blacktriangleright n$ is false. However, this is irrelevant, because, either way, the left side of the conditional will be false because, for m, we get $m \sqsubset s \wedge ... \neg m \sqsubset s$, and likewise for n, we get $n \sqsubset s \wedge ... \neg n \sqsubset s$. Accordingly, if either model fails to satisfy A7, then so does the other model.

A8. To satisfy A8,

$$\forall r \forall s\ ((Cr \wedge\ \forall x[(x \neq r \wedge x \neq s) \Rightarrow\ (r \blacktriangleright x \Leftrightarrow s \blacktriangleright x)]) \Rightarrow r = s)$$

for any two *distinct* phenomena that cause anything, there must exist a third phenomenon, x, distinct from the othe two, which is caused by one but not by the other. Since x is *relative*, this will be the same in both models.

A9. To satisfy A9,

$$\exists x\ (\neg Ax \wedge\ Cx)$$

there must exist a causal chain of at least three distinct phenomena, where the latter two are *relative*. Hence, if this exists in one model, it must exist in the other.

A10. To make a model fail to satisfy A10,

$$\forall r \forall s\ ((\neg Ar \wedge\ r \sqsubset s) \Rightarrow \neg As)$$

there must exist an Absolute phenomenon, s, which has some part, r, that is relative. Since both M and N have identical part relationships, if one fails to satisfy A10, then so does the other.

This proves the Causal Equivalence Principle for the theory which is identical to TAP except that it excludes A4.

Now, to see that the Causal Equivalence Principle holds for the whole of TAP, suppose that one model, either M or N, satisfies all the axioms, including A4. Since the Causal Equivalence Principle holds for all axioms except A4, the other model therefore also satisfies all the axioms, except perhaps A4. In particular, both M and N satisfy A10.

A4. To satisfy A4,

$$\forall r \forall s \forall t\ ([r \blacktriangleright t \wedge\ s \sqsubset t \wedge \neg As \wedge\ r \neq s] \Rightarrow r \blacktriangleright s)$$

any *self-causing* phenomenon must cause all of its *relative* parts. Since Sm, m must cause all of its relative parts. But since both models satisfy A10, *no* such self-causing, Absolute phenomenon (including m) has *any* relative parts, which means A4 is satisfied by both models.

This completes the informal proof of the Causal Equivalence Principle. Thus, in TAP, it makes no difference whatsoever whether any Absolute phenomenon is self-causing or causeless.

Neither A4 nor A10 is needed in the variation of TAP which uses the revised A1, so the Causal Equivalence Principle applies to it as well. However, if one finds useful a variation which includes A4 but not A10, the Principle might not apply. This can be remedied by again modifying A4 so that it does not require a self-causing phenomenon to cause its parts:

$$\forall r \forall s \forall t\ ([r \blacktriangleright t \underline{\wedge r \neq t} \wedge s \sqsubset t \wedge \neg As \wedge r \neq s] \Rightarrow r \blacktriangleright s)$$

This is the same as A4 except the underlined conjunct. The added requirement that $r \neq t$ resolves the issue.

The Faith of Atheism

"[To an atheist] the universe is the most exquisite masterpiece ever constructed by nobody."

—Gilbert Keith Chesterton (1874-1936),
quoted by Martin Gardner [4 at pp. 238-239]

What Is Faith?

Any mathematical theory is based on axioms which cannot be proven. These axioms must be accepted *on faith*. Analogously, any non-mathematical assertion, worldly or abstract, may be justified through logical reasoning, but the reasoning can only go so far until reaching the basic assumptions, and these too must be accepted *on faith*. Therefore, faith (sometimes called intuition) is the basis of all reasoning and the foundation of the intellect.

Theistic faith is just the acceptance of unproven *theological* assumptions. On the basis of such assumptions, a religious person accepts God's existence and divine (or supernatural) qualities.

Now, in the light of the proof of T8, at least for anyone who understands the proof, *theological assumptions are no longer needed* to believe in God's existence, omnipotence, Absoluteness, and uniqueness!

There are countless popular but unproven theological assumptions, perhaps the most important of which states *it is possible to relate with God on a personal level*. This concept of establishing and maintaining a personal relationship with God is for many theists the *quintessential* aspect of faith.

Through *G* Theory, a mathematical "communion" *establishes a relationship with God via the intellect*. For those who seek to relate to God through the heart rather than through the intellect, this abstract connection may seem counter-intuitive or superfluous. But for the scientists, philosophers, and seekers of knowledge, devotion to Truth must be a sincere commitment. This devotion binds the heart and intellect, allowing on the one hand a deeper understanding of spiritual concepts, and on the other, a healthy skepticism. Materialists will likely scoff at the idea of Absolute Truth – how could it be material? But they must now reckon with *G*, which by any realistic interpretation *transcends* the familiar material world, just as Ω transcends arithmetic.

Is God Impersonal or Personal?

There are many arguments for both the existence and non-existence of God, with varying definitions or qualifications for Godhood. Myriad philosophies and theologies will accept some of these arguments and refute other ones. While many theologies claim that these arguments validate God's existence *rationally*, there are also theologies maintaining that the arguments are all moot because God can only *truly* be known and experienced through *faith* (this may just mean accepting some dogma on faith, such as a religious scripture).

Similarly, some atheistic philosophies will use arguments to validate God's non-existence rationally, whereas others (usually considered *agnostic*) maintain that all rational arguments concerning God's existence are inconclusive.

The most powerful tool of rational argument ever created, First-Order Predicate Logic, proves conclusively the positive result: that God *does* exist. But the question remains whether God is personal or impersonal, *i.e.* is it possible to relate with God on a personal level?

While the vast majority of religions promise a personal God, there are also many philosophies which characterize God as entirely impersonal. For example there is the idea that God initially created the universe, but left it alone thereafter. This would mean that all relationships are within the creation, and it is impossible to relate to this elusive God. Other examples include monism, which is the literal belief that everything is one (the extreme case of the idea that everything is connected or related), and the slight variation pantheism, which says that everything is God. Under either assumption, since everything is the same (everything literally is God), there cannot be any distinction between a relationship with God and a relationship with something other than God.

For those who *reluctantly* abandon atheism due to the evident soundness of T8, this concept of an impersonal creator will undoubtedly be their only consolation. Others will most likely cling to their atheism and maintain conviction that T8 is somehow unsound.

The Nine Schools of the Atheist Faith

Often atheism is intended to mean a *lack* of religious beliefs: *not* believing in God is different from believing *there is no* God. In order to defend the assertion "There is no God" there must be some logical argument (which in practice may be as simple as declaring that a particular idea of "God" is ill-defined). A lack of beliefs is a lack of assertions, which requires no defense. However, to deny T8 is to assert that it is false, and this leads to bizarre consequences. By denying T8, the atheist must either abandon logic altogether, or accept some bizarre cosmological alternative *on faith*.

T8 can only be false if some axiom is false. Within the 9 atheistic "Schools" enumerated below, every logically possible way for T8 to fail in proving God's existence is examined. Each School is shown to necessitate absurdities, falsities, or just generally bizarre beliefs.

S1. Nihilism.

Nothing exists. (And therefore, God doesn't exist.) This is a preposterous assertion: everything is a counterexample.

S2. Nescience.

If you do not understand the proof, then you could deny its validity. Similarly, if you do not understand the proof of the Pythagorean Theorem, you could deny its validity as well. However, when complete understanding is attained, denial becomes extremely difficult or impossible.

S3. Mathematics does not represent reality.

Anyone who has actually bothered to study *G* Theory is probably not willing to reject mathematics. Such would be an extremely strong, bizarre rejection.

S4. Causation doesn't exist.

This is a strong, bizarre assertion. It may be likened to abandoning science altogether, or at least the very large and essential portion of science which claims that causal relationships exist.

S5. Time is circular, so causation is non-linear.

The concept of "eternal recurrence" states that ultimately, the universe repeats infinitely, meaning if you go far enough into the "future", you will effectively arrive in the "past". This fascinating concept is definitely bizarre and highly dubious, and it appears to indicate circular causation, which contradicts A3.

S6. There are multiple Absolute, omnipotent phenomena.

This position becomes possible by denying A8, Equipotency. It is certainly a strong, bizarre assertion; but it sounds more polytheist than atheist. An absurd alternative, rather than denying A8, would be to deny the existence of relative phenomena, contradicting A9. Without any relative phenomena, no phenomenon could cause anything (other than itself), yet ironically, everything would be (relatively) omnipotent. Since A8 only applies when a phenomenon causes something *else* (other than itself), A8 is not violated in this case.

S7. An omnipotent phenomenon exists, but it is not Absolute.

This is so absurd, it seems hardly worth mentioning! If an omnipotent phenomenon is relative, then it automatically violates SCEP; additionally, it would mean that a second omnipotent phenomenon exists which causes the first – but then if neither is Absolute, each must cause the other, violating A3.

S8. Absolute phenomena exist, and A1 is true, but A6 is unfair because it combines a possibly infinite number of phenomena into an omnipotent whole.

In reality, the effect of the *sum* of all Absolute phenomena is apparent, because clearly a complex universe of relative phenomena does exist. It would be bizarre to assert that Absolute phenomena cannot coexist as a whole, while simultaneously acknowledging this sum effect which comprises a whole, which is the universe.

Or instead, allowing A6 to be true, imagine that even though (by A1) its parts collectively cause everything, *G* fails to be omnipotent because A7 is false – but how could A7 be false? Has a counterexample ever been observed where the whole does not cause what the parts cause?

S9. Absolute phenomena do not exist, so A1 and A5 are both false.

As elaborated in the section on the revised A1, rejecting A1 requires a double standard: none of the links in an infinite chain are allowed to be Absolute, but the infinite chain is somehow exempt from being Absolute even though it cannot have a cause (other than itself, of course). This double standard appears to be entirely contradictory and unjustifiable.

Furthermore, denying Absolute phenomena also means that the universe does not exist, because only something Absolute could cause everything in the universe (and of course if nothing causes the universe, then the universe itself is Absolute). It also means that causation cannot exist (as in *H* from A5), because (as T2 demonstrates) whatever causes causation must similarly be Absolute (and of course, if nothing causes *H*, then *H* is Absolute).

Therefore, in order to deny that T8 soundly proves God's existence, the atheist must have faith in the truth of at least one of S1-S9. But each of these Schools posits either a falsity or a strong, bizarre assertion. This seems at odds with the spirit of atheism and agnosticism, which typically *strive to avoid both falsities and bizarre assertions at all costs*!

Conclusion

The necessary existence of an Absolute phenomenon was demonstrated in the justification of the revised A1. The possibility of multiple Absolute phenomena was explored. The existence and uniqueness of an Absolute, omnipotent God, as expressed in T8, was proven in 2 different ways: first using all of TAP, and then using the variant theory which includes the revised A1 and excludes some of TAP's axioms. The parts of God are precisely the Absolute phenomena (other than God); but if God is the only Absolute phenomenon (as is the case in Version 1) then God has no parts.

Causeless phenomena were allowed, but the Causal Equivalence Principle showed the insignificance of attempting to distinguish between causeless and self-causing phenomena.

All 13 axioms from TAP, its variant, and Version 1 were collectively shown consistent by a simple model.

The consequences of denying the existence of our formally defined God were examined thoroughly. While the justifications for the axioms were all consistent and more or less intuitive, every conceivable refutation of T8 led to either a contradiction, an absurdity, or a bizarre, counter-intuitive circumstance. Evidently the God of T8 is easily justified through science, mathematics, and intuition, whereas atheism appears to require some supernatural leap of faith.

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